Polynomial kernels for hitting forbidden minors using constant treedepth modulators

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Computations on Networks with a Tree-Structure: From Theory to Practice

joint work with Bart M. P. Jansen

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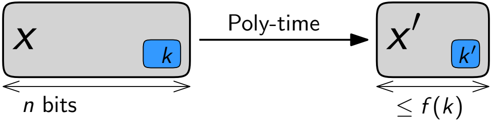
Polynomial-time preprocessing

A kernelization for a parameterized problem is a polynomial-time algorithm that transforms input (x, k) to (x', k') such that

- (x, k) is a yes-instance if and only if (x', k') is a yes instance
- $|x'| \leq f(k)$ and $k' \leq f(k)$



Goal

- Obtain polynomial kernels for a wide variety of problems
- Small parameter k

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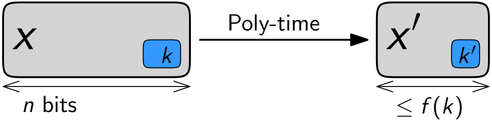

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Parameterizations

One of the most common parameterizations: solution size

- Good when large input asks for small solution
- Does not give good bounds when solution size $\approx$ input size

Focus on structural parameters instead

- Good when input has simple structure

Meta theorems

Courcelle's Theorem

All problems that can be expressed in MSOL on graphs, can be solved in linear time on graphs of bounded treewidth

Obtain similar results for kernelization?

- DOMINATING SET has no polynomial kernel when parameterized by VERTEX COVER [Dom et al. ICALP'09]
  - Large structural parameter
- But is easy to express in most types of logic

Focus on another very general problem

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Focus on another very general problem

$\mathcal{F}$-Minor-Free Deletion

Let $\mathcal{F}$ be a set of connected graphs

Definition

Input	A graph G and integer k
Question	Does there exist $S \subseteq V(G)$ with $|S| \leq k$, such that no graph in $\mathcal{F}$ is a minor of $G - S$?

We sometimes say S breaks $\mathcal{F}$

Generalizes many problems

- $\mathcal{F} = \{$ $\}$: Vertex Cover
- $\mathcal{F} = \{$ $\}$: Feedback Vertex Set
- $\mathcal{F} = \{$, $\}$: Making a graph planar by vertex-deletions

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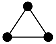
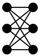
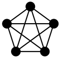
- $\mathcal{F} = \{$  $\}$: Feedback Vertex Set
- $\mathcal{F} = \{$  ,  $\}$: Making a graph planar by vertex-deletions

Structural parameters

Many problems are easy for some simple graph class $\mathcal{G}$

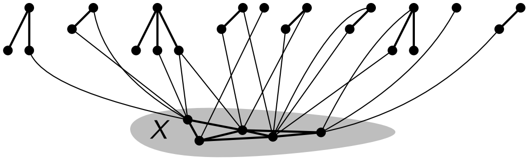
- Trees, cliques, paths, independent sets, forests, ...

Treewidth, pathwidth, cliquewidth, ...

- VERTEX COVER has no polynomial kernel [Bodlaender et.al. 2009]

Number of vertices we need to remove until $G \in \mathcal{G}$

- Sometimes allows for polynomial kernels
- Also called a modulator



Parameter: modulator to constant treedepth

X is a treedepth-η modulator when $\mathrm{td}(G - X) \leq \eta$

- η is considered a fixed constant

Parameter: $|X|$ for optimal X

$\mathcal{F}$-Minor-Free Deletion is easy on graphs of constant treedepth

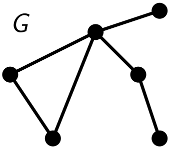
- Polynomial-time solvable

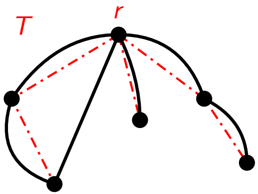
Treedepth

td(G) is the minimum depth of any treedepth decomposition

- Tree T on all vertices of G
- Any edge in G is between children/ancestors in T

Example of treedepth 3





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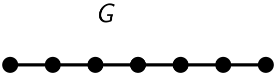

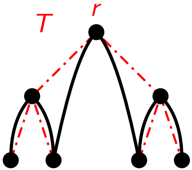

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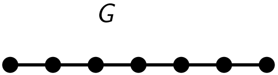

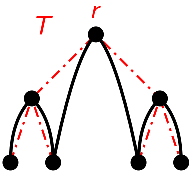

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Previous work

Parameterized by treewidth-η modulator [Jansen,Bodlaender STACS'11]

- Polynomial kernel for Vertex Cover for $\eta = 1$
- No polynomial kernel for Vertex Cover for $\eta \geq 2$

Parameterized by treedepth-η modulator

- Polynomial kernel for Vertex Cover [Bougeret,Sau IPEC'17]
  - Kernelization of Feedback Vertex Set left open

Results

Let $\mathcal{F}$ be a set of connected graphs

Theorem

$\mathcal{F}$-MINOR-FREE DELETION has a polynomial kernel parameterized by a treedepth-η modulator

- Kernel of size $O(|X|^{g(\eta,\mathcal{F})})$ for some function g
- Resolves the question about FVS

Lower bound

VERTEX COVER parameterized by a treedepth-η modulator has no kernel of size $O(|X|^{2^{\eta-4}-\varepsilon})$, unless $NP \subseteq coNP/poly$

- g is exponential in η, and this cannot be avoided

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Kernel for $\mathcal{F}$-Minor-Free Deletion

Kernel: main ingredients

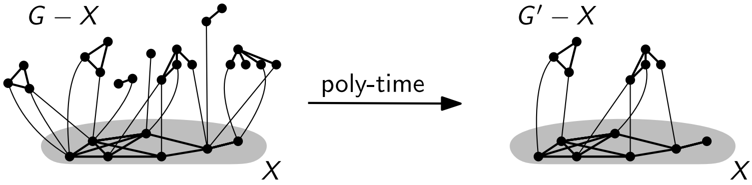
Given G with modulator X

- Reduce the number of connected components of $G - X$
- Apply induction on η to obtain kernel

Lemma

There is a polynomial-time algorithm that transforms G into induced subgraph G', and returns an integer Δ such that

- $\text{opt}(G') + \Delta = \text{opt}(G)$
- $G' - X$ has at most $|X|^{O(1)}$ connected components



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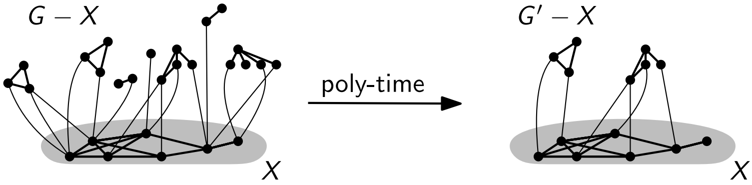

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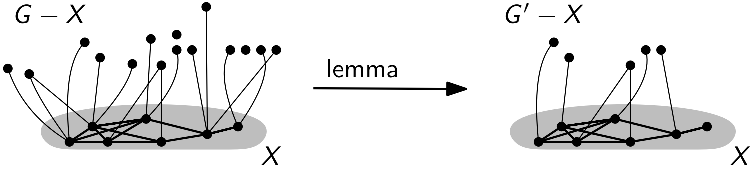
Theorem

$\mathcal{F}$-Minor-Free Deletion has a polynomial kernel parameterized by a treedepth-η modulator

Base case: $\eta = 1$

Every connected component of $G - X$ is a single vertex

- Given G and budget k, apply lemma to obtain G' and Δ
- Let the kernel be G' with budget $k - \Delta$
- G' has at most $|X| + |X|^{O(1)} = |X|^{O(1)}$ vertices

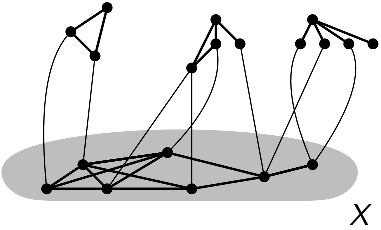


Kernel: using Lemma

Step: $\eta > 1$

Apply lemma, find G' with only few components in $G' - X$

- For each component of $G' - X$, select root vertex r
  - Can be found efficiently



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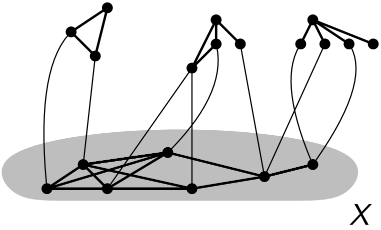

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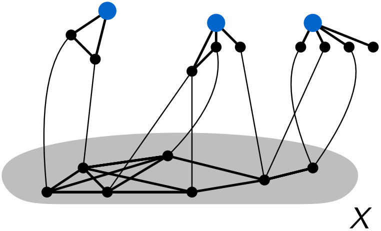

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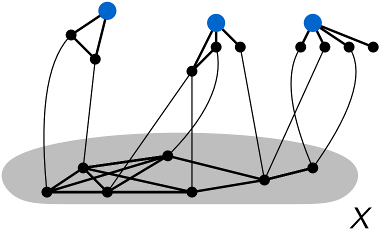

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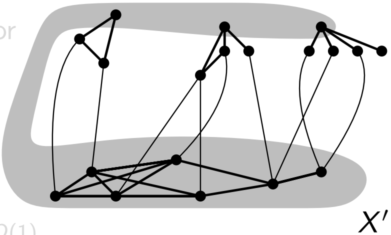
- $|X'| = |X|^{O(1)}$



Apply IH with X'

- Gives kernel of size $|X'|^{O(1)} = |X|^{O(1)}$

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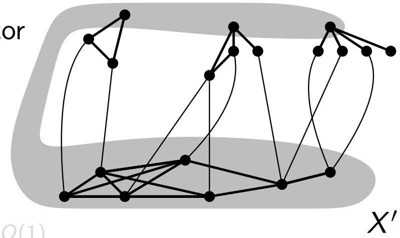

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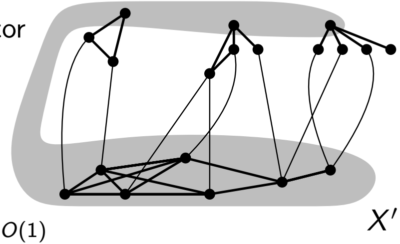

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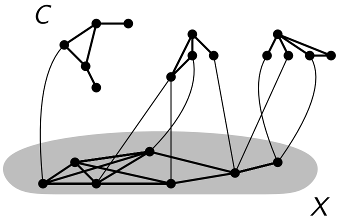
Towards a proof sketch of the Lemma

Example: $\mathcal{F} = \{K_3\}$.

- FEEDBACK VERTEX SET (abbreviated as FVS)

Try to remove connected components of $G - X$

- Exists an FVS in $G[C]$ disconnecting C from X
  - Remove C
  - Reduce budget by opt(C)



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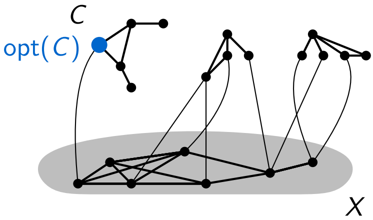

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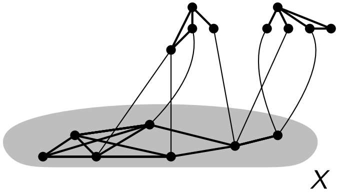

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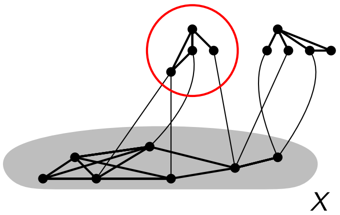


Solutions in G: property

Let S be an optimal Feedback Vertex Set in G

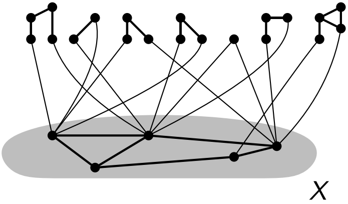
- Also works if S is an $\mathcal{F}$-deletion

Lemma

There exist $\leq |X|$ components C in $G - X$ such that

- S is not locally optimal in C

Any optimal FVS is locally optimal for most components!



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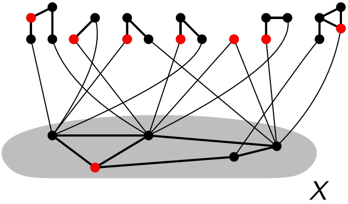

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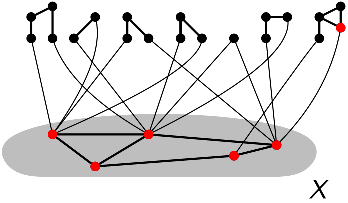

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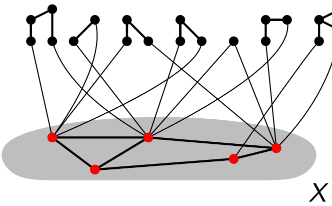
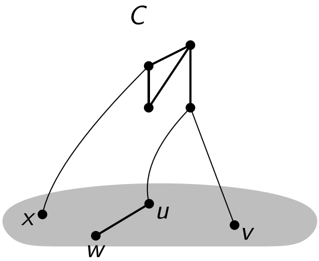


Removing components of $G - X$

Consider which $x \rightsquigarrow x'$-connections are made by C for $x, x' \in X$

- After removing some optimal FVS

No optimal FVS breaks $u \rightsquigarrow v$ in C



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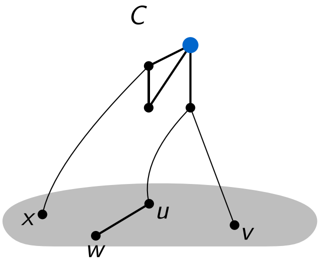
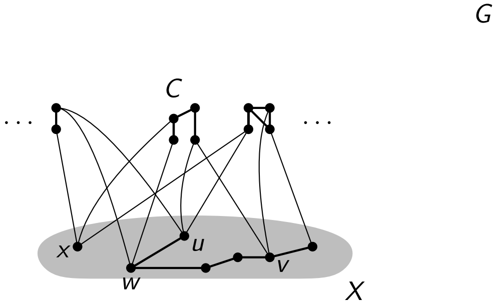
Some optimal FVS breaks $x \rightsquigarrow v$ and $x \rightsquigarrow u$ in C



Removing components of $G - X$

Suppose opt(C) never breaks $u \rightsquigarrow v$, $v \rightsquigarrow w$,$u \rightsquigarrow w$,$w \rightsquigarrow x$,. . .

- Select a number of representative components
  - Mark $|X|^c$ other components that do not break $u \rightsquigarrow v$
  - Mark $|X|^c$ other components that do not break $v \rightsquigarrow w$
  - . . .
- Remove C, decrease budget by opt(C)



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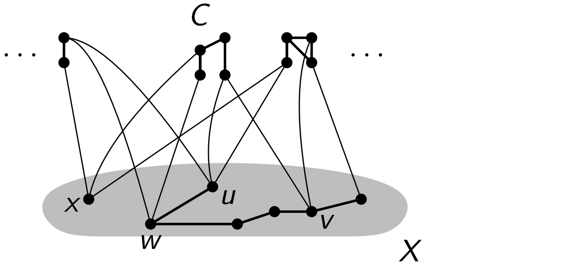
- If S is a FVS in G', $S \cup S_C$ is a FVS in G



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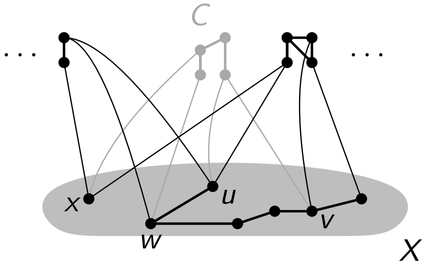

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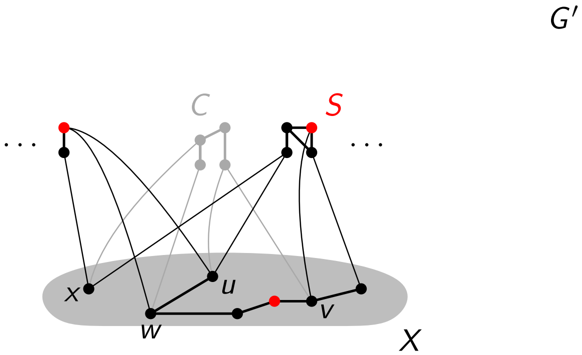

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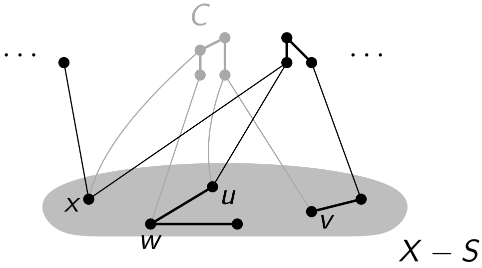
$G' - S$



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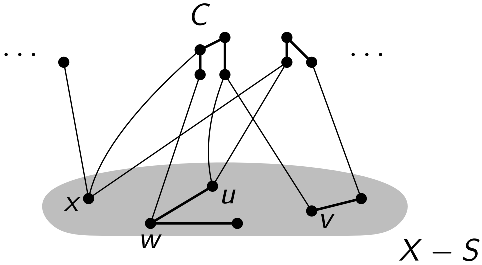
$G - (S \cup S_C)$



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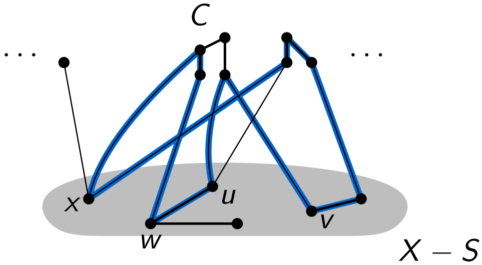
$G - (S \cup S_C)$



Removing components of $G - X$

Suppose C was removed by this rule

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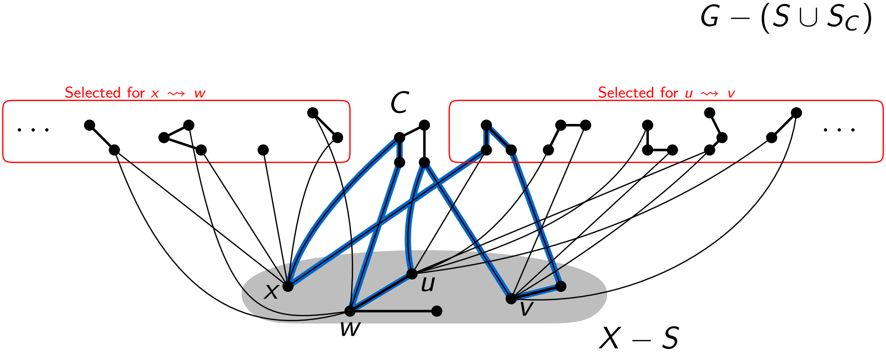

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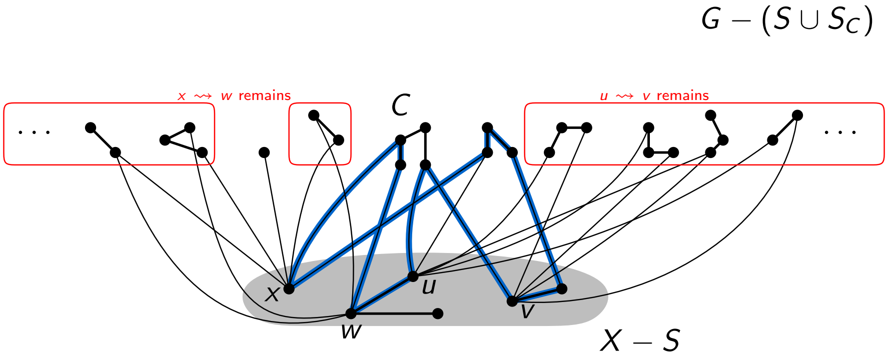

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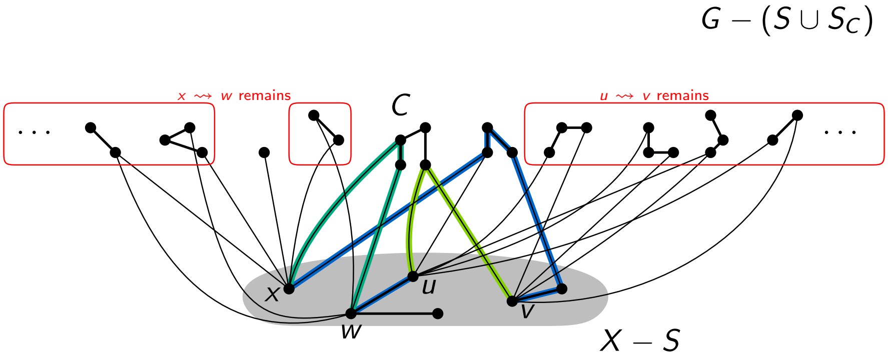

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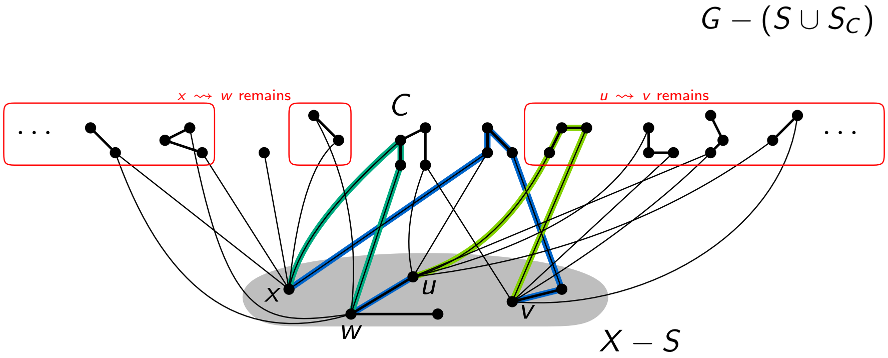

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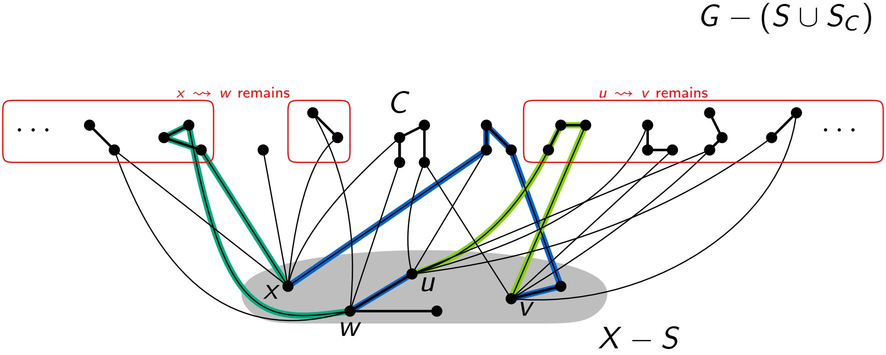

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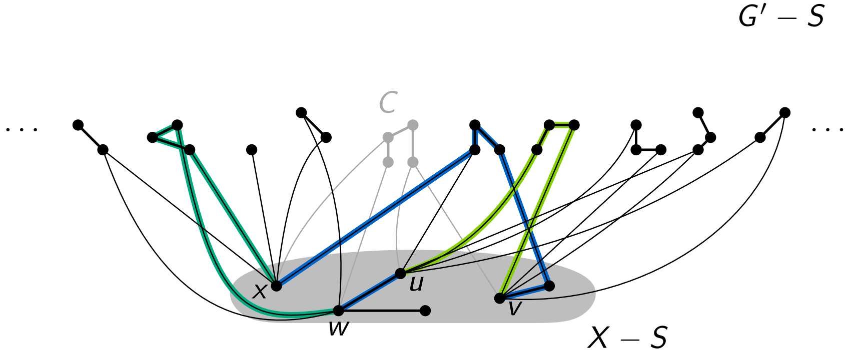

Removing components of $G - X$: rules so far

Cases we handled

- Some optimal FVS in C breaks all connections of C to X
- Any optimal FVS in C leaves connections $u \rightsquigarrow v$,$v \rightsquigarrow w$,...
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Any other options to consider?

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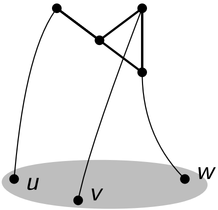

For each set L of possible connections, indicate if there exists an optimal FVS in C that breaks all connections in L.

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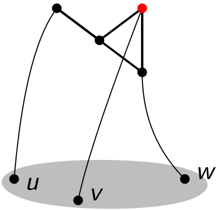

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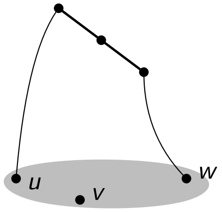

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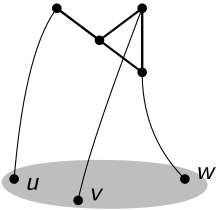

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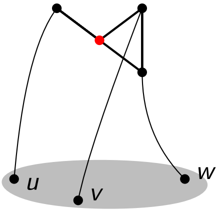

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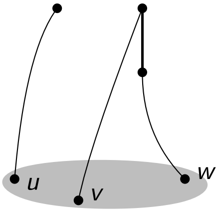

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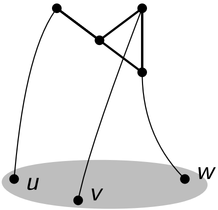

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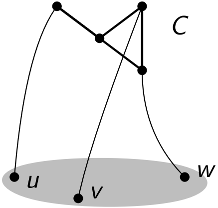
Classifying components

All relevant information about C:

$L =$	broken by optimal FVS in C?
$\{u \rightsquigarrow v\}$	yes
$\{u \rightsquigarrow v, u \rightsquigarrow w\}$	yes
$\{u \rightsquigarrow v, u \rightsquigarrow w, v \rightsquigarrow w\}$	no
...	...



Towards a reduction rule

Behaviour of components described sets L that cannot be broken

- For each L, mark poly($|X|$) representative components
- Remove unmarked components

But there are many sets to consider

- Number of subsets of $X \times X$
- Exponentially many such sets

Ideally: only need to consider sets of constant size γ

- Dependent on $\mathcal{F}$ and η

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- What if a FVS cannot break very large set L
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Main effort in our paper is showing that this does not happen

- Constant size witness

Main Lemma (sketch)

Let L be a set of pairs from X, let C be a graph of constant treedepth. If no optimal FVS in C breaks the connections between all pairs in L, then there exists $L' \subseteq L$ such that

- $|L'| \leq \gamma$
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This leaves polynomially many components in $G - X$.

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Treewidth versus treedepth in main lemma

Rephrased for VERTEX COVER

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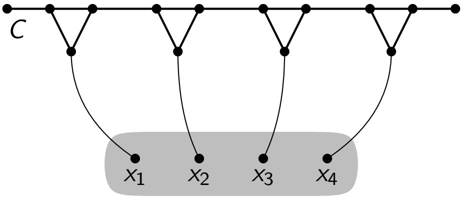
The use of treedepth and not treewidth is essential



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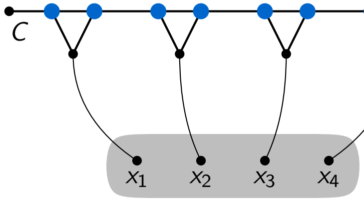

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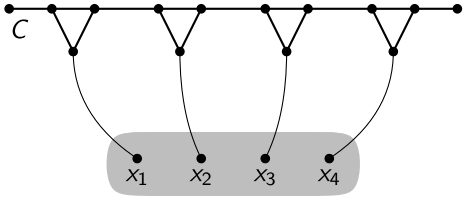

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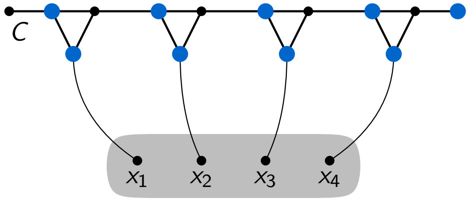

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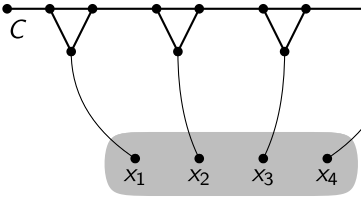

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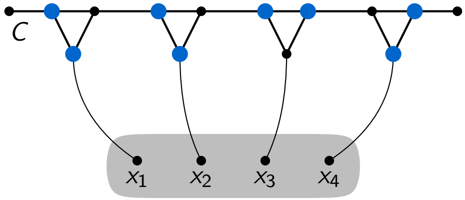

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Conclusion

$\mathcal{F}$-Minor-Free Deletion parameterized by a treedepth-η modulator has a polynomial kernel

- Graphs in $\mathcal{F}$ must be connected

Future work

Find the most general graph class $\mathcal{G}$ such that

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Thank you